A quantitative and qualitative comparison between the empirical mode decomposition and the wavelet analysis

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Abstract. In this paper we compare two techniques developed in recent years to overcome the limitations of conventional approaches, the wavelet transform and the empirical mode decomposition. The wavelet analysis decompose data through multiple scales using a previously selected basis from many available wavelet function families, moreover it has a fast discrete algorithm which is computationally less complex than the Fast Fourier Transform. In the empirical mode decomposition the basis is extracted from the data itself, thus it is optimal for that given data set and provides a more detailed view of the internal dynamics of the process, however it is computationally intensive. First we present a brief theoretical background of both techniques. Then we review some experimental results with complex and chaotic time series.

1. Introduction

In order to obtain useful information from complex data is often desirable to separate the data into its basic components such that those components are easier to examine and give a more meaningful representation of the phenomenon.

Unfortunately the traditional approach – the Fourier analysis and its derivatives – assumes that the data is linear and stationary, this assumption leads to a very poor resolution of the embedded structures of the phenomenon in this new representation, since most of the significant phenomena are non-stationary and present non-linear effects to some degree.

Wavelet analysis is now a well established theory based in the multiresolution concept where a prototype function with compact support is dilated and contracted to see the general behavior and the fine detail in the data

With this approach we can overcome the limitations with non-stationary data that Fourier analysis presents and obtain a time-scale-energy representation that has been very useful in many areas.

The Empirical Mode Decomposition (EMD) is based on the concept of extracting the intrinsic mode functions (IMF) from the data. Such functions have a meaningful instantaneous frequency and in consequence a well behaved Hilbert transform.

The EMD is defined by an algorithm rather than an analytical function, however given that the basis is extracted from the data itself, it is adaptive and in addition to overcome the non stationary limitations, it provides more detail for the non-linear effects in the data set because it extracts the oscillatory modes inherent to the phenomenon.

2. Wavelet analysis

The Continuous Wavelet Transform (CWT) is defined as

$$F_{\psi}(a,b) = \langle f, \psi_{a,b} \rangle = \frac{1}{a} \int_{-\infty}^{\infty} f(t) \psi^{*}\left(\frac{t-b}{a}\right) dt \tag{1}$$

Where ψ is a prototype function called "mother wavelet", b is the location of the analysis window and a is the scale. Such function must fulfill certain requirements such as having finite length and energy, and it has to be an oscillatory function that is well localized both in time and frequency. A wavelet prototype is often called a time-frequency atom.

When we deal with discrete time functions, we can calculate the inner product of the wavelet basis and the data using numerical methods, however it is computationally intensive and it yields redundant data, for that reason there was a need to found a way to construct a discrete transform with a fast algorithm.

Fortunately, a close relationship between wavelets and filterbanks was found. If we define two functions such that

$$\phi(x) = \sum_{n=-\infty}^{\infty} c_n \phi(2x-n) \tag{2}$$

$$\psi(x) = \sum_{n=-\infty}^{\infty} (-1)^n c_{-n+1} \phi(2x-n) \tag{3}$$

a low-pass, perfect reconstruction FIR filter h_0 can be constructed as described in [1].

Once the low pass filter is constructed, using (4) we obtain a quadrature mirror filterbank (QMF), such filterbank is used for analysis and synthesis of the data.

$$h_1(n) = (-1)^n h_0(L-n-1) \tag{4}$$

The Discrete Wavelet Transform (DWT) is then defined as

$$y_h(k) = \sum_{k=-\infty}^{\infty} x(n) \cdot h_0(2k-n) \tag{5}$$

$$y_l(k) = \sum_{k=-\infty}^{\infty} x(n) \cdot h_1(2k - n) \tag{6}$$

The function $y_h(k)$ is called the detail coefficients and it is obtained filtering the original signal with the high pass filter and then downsampling in a dyadic fashion. The function $y_l(k)$ is called the approximation coefficients, it is obtained with the low pass filter, each one of them can be further decomposed with the same filterbank in octave bands or in an arbitrary tree.

This result is of great importance because it means that every orthogonal wavelet basis has an associated filter and thus, a fast discrete transform, furthermore it is also shown that wavelets can be constructed from FIR filters although not every filter can result in a wavelet.

The main drawback in the wavelet analysis is that once a basis is selected, it is used during the whole process, and we cannot assume that a single basis is the best for all the data – especially in non-linear cases – and while the resolution is acceptable, sometimes a better resolution is desirable.

3. Empirical mode decomposition

The empirical mode decomposition was introduced in [3] by Huang et al to obtain an analysis method that was complete, orthogonal, local and adaptive from where time-frequency scales were extracted. The principle of this technique is to decompose a signal x(t) into a sum of intrinsic mode functions such that:

$$x(t) = \sum_{i=1}^{n} imf_i(t) + r_n(t) \tag{7}$$

An intrinsic mode function is a function that satisfies two conditions (1) have the same numbers of zero crossings and extrema; and (2) are symmetric with respect to the local mean. The first condition is similar to the traditional narrow band requirements for a stationary Gaussian process. The second condition modifies the classical global requirement to a local one; it is necessary to eliminate unwanted fluctuations from the instantaneous frequency that asymmetric wave forms introduce.

With this definition, an IMF is not restricted to a narrow band signal, and it can be both amplitude and frequency modulated. This provides more flexibility as one embedded oscillation mode is allowed to have more than one component.

The algorithm for the decomposition is as follows

1. Initialize r(t) = x(t), i = 1
2. Extract the i-th IMF:
   - (a) Initialize h0(t) = ri(t), j = 1
   - (b) Extract the local minima and maxima of hj-1(t)

(c) Interpolate the local minima and maxima by a cubic spline to form upper and lower envelopes of hj-1(t)
(d) Calculate the mean mj-1(t) of the upper and lower envelopes of hj-1(t)
(e) hj(t) = hj-1(t) – mj-1(t)
(f) If stopping criterion is satisfied then
set imfi(t) = hj(t)
else go to (b) with j = j + 1

3. ri(t) = ri-1(t) – imfi(t)
4. If ri(t) still has two extrema, then
go to 2 with i = i + 1
else the decomposition has finished and ri(t) is the residue.

Step 2 is known as the sifting process, and has two effects: eliminates riding waves; and smoothes uneven amplitudes. Unfortunately, the second effect, when carried to the extreme, could destroy the physically meaningful amplitude fluctuations. To guarantee that the IMF components retain enough physical sense of both amplitude and frequency modulations, in the stopping criterion we have to limit the size of the standard deviation between two consecutive sifting results, the suggested value in the literature is 0.3.

4. Experimental results

We analyzed a set of 30 time series with both techniques, here we present detailed results for one of them, the time series are measurements from El Niño phenomenon of complex nature (Figure 1). All time series consist of one thousand samples.

The complete list of the employed time series and their classification according to [6] and [7] is shown in Table 1.

Table 1. Time series used for the experiments

Time Series	*Nature*	*Time Series*	*Nature*
Sine	periodic	Dow Jones	complex
Vanderpol	periodic	Kobe	complex
Qperiodic2	quasiperiodic	ECG	complex
Qperiodic3	quasiperiodic	EEG	complex
Mackey-Glass	chaotic	ASCII	complex
Logistic	chaotic	El niño	complex
Lorenz	chaotic	HIV DNA	complex
Rossler	chaotic	Human DNA	complex
Ikeda	chaotic	Lovaina	complex
Henon	chaotic	Plasma	complex
Cantor	chaotic	Primes	complex
Tent	chaotic	SP500	complex
A1	complex	Star	complex
D1	complex	Brownian motion	complex
Laser	complex	White Noise	complex

The DWT was computed obtaining six decomposition levels using the 'DB8' wavelet. The EMD was computed until no extrema were found in the residual.

Two interesting cases were the prime number and the Lovaina time series. The prime number time series consists in the distance from one prime number to the next, and the Lovaina time series was generated from ASCII data for the Lovaina University contest.

In the prime number case, there were contiguous identical maxima, consequently the spline interpolation algorithm returned with error.

In the Lovaina case, during the sifting process, the extracted functions never fulfilled the 0.3 standard deviation stopping criterion, so it iterated indefinitely, when the criterion was relaxed to 0.35, the EMD went without trouble and the original time series were reconstructed from the IMFs with no significant distortion

In Figures 2 and 3 we can appreciate how the EMD and the DWT decompose the data going from fine details to a coarse approximation, but the oscillations in each level of decomposition are more contoured in the IMFs.

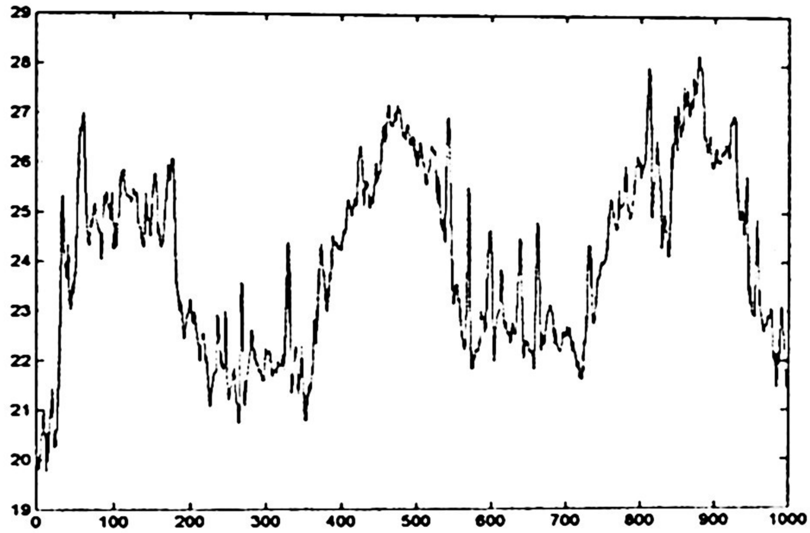

Fig. 1. El Niño time series

Also we can notice how the number of IMFs increases with the intricacy of the data. In Table 2 lists the number of extracted IMFs using the EMD algorithm with each time series.

Unlike the discrete wavelet transform which is dependant on the amount of samples in the time series, the number of intrinsic mode functions extracted from the data with the empirical mode decomposition, is dependant only from the dynamics of the phenomenon. Even the most intricate time series resulted in no more than 16 intrinsic mode functions, and the average number of IMFs is 12.

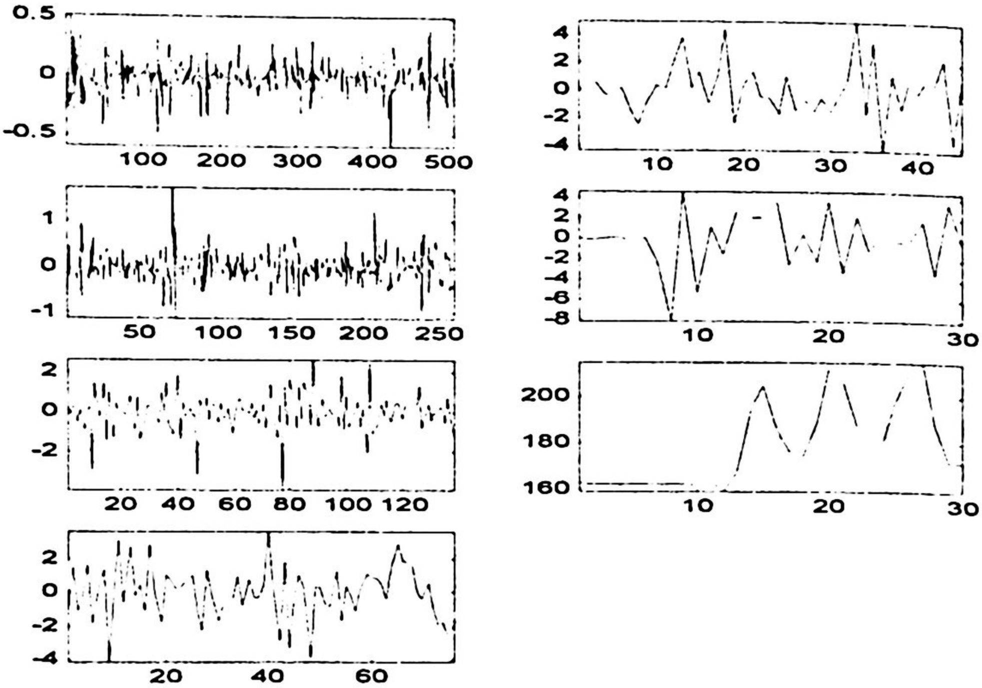

Fig. 2. El Niño DWT detail and approximation coefficients

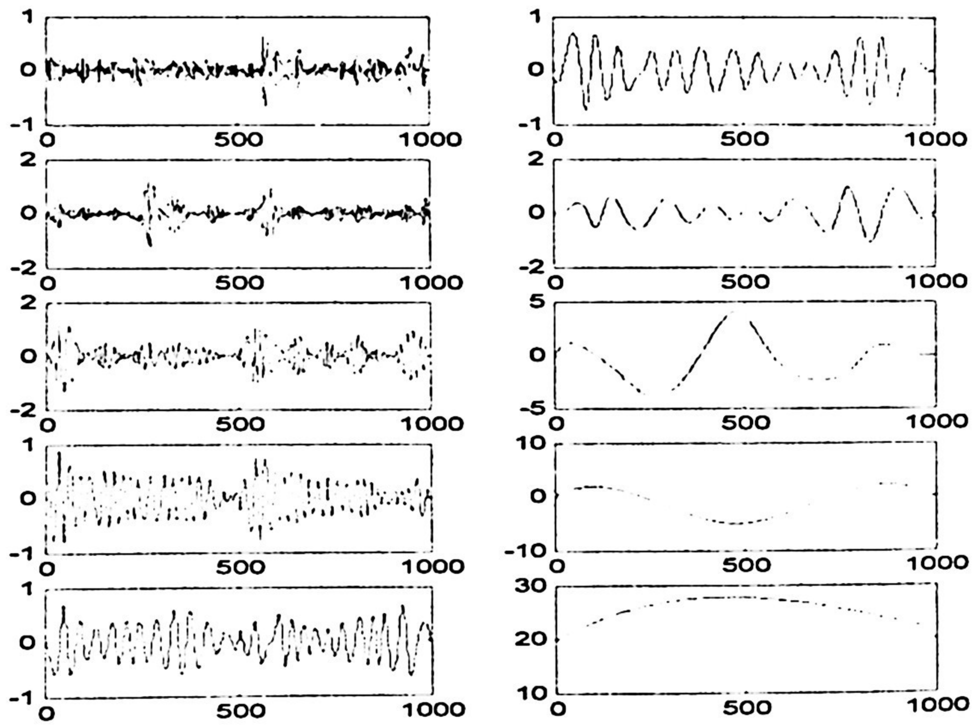

Fig. 3. El Niño IMFs and residual resulting from the EMD

Table 2. Number of extracted IMFs

Time Series	*IMFs*	*Time Series*	*IMFs*
Asciitxt	15	logistic	14
brownian	10	lorentz	7
Cantor	14	lovaina	10
D1	12	mackey	14
Djones	12	plasma	16
Ecg	9	qperiodic2	9
Eeg	13	qperiodic3	12
Elnino	10	rossler	8
henon	16	SP500	15
hivdna	13	Sine	8
humandna	9	Star	13
ikeda	15	Tent	16
kobe	15	vanderpol	6
laser	15	whitenoise	15

Another interesting property of the EMD is that the oscillatory modes it finds may have certain physical meaning, not jus a band of frequencies than comprise the signal and the residual can be interpreted as a trend of the time series.

In the Hilbert spectrum in Figure 4 it is also shown how the energy is more spread in the wavelet analysis while with the IMFs the instantaneous frequencies are very accurate and thus is easier to understand the behavior of the phenomenon as the frequency components at a given instant are evident.

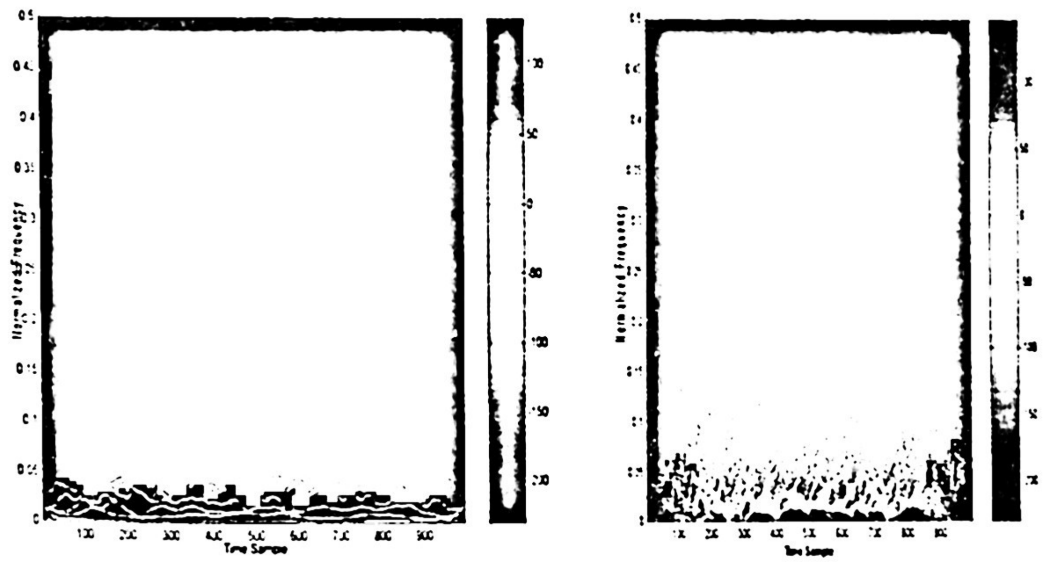

Fig. 4. (a)El Niño IMFs Hilbert Spectrum (b) El niño CWT Hilbert Spectrum

In Figure 5 we can observe that both techniques can be useful to remove certain components from data. First we added Gaussian noise to the ECG time series, to reduce the noise components we disregarded the first five detail levels in the DWT case, and the first six IMFs of the EMD. After reconstruction, similar results were obtained, the EMD obtained better results reconstructing the main peak of the ECG (the QRS complex) but in the last part, the signal reconstructed from the DWT is less distorted.

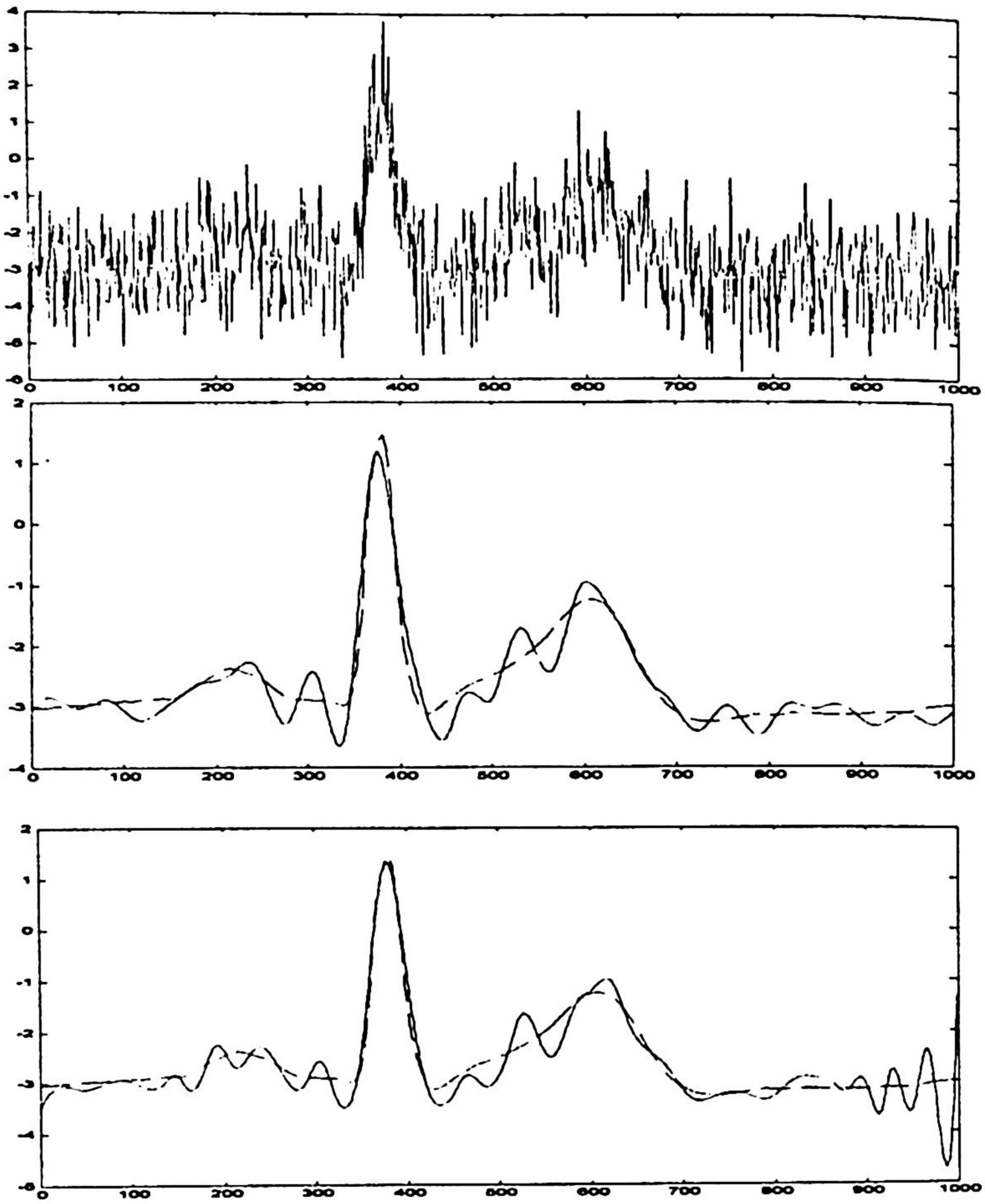

Fig. 5. (a)ECG with additive Gaussian noise, parameters μ=0, σ=1 (b) ECG denoised discarding detail levels (c) ECG denoised discarding IMFs

Finally in Table 3 we can compare the signal to noise ratio (SNR) obtained from the original data and the reconstructed data with both techniques, as we can see, both offer excellent reconstruction and the error could be attributed to the floating point operations but the EMD reconstruction offers much lower distortion probably because the DWT losses energy in the downsampling and upsampling stages

Table 3. SNR for reconstructed data.

Series	*EMD*	*DWT*	*Series*	*EMD*	*DWT*
Asciitxt	315.1785	236.3873	logistic	313.2426	235.9292
brownian	315.6049	225.7096	lorentz	317.2882	222.7929
Cantor	315.414	229.9274	lovaina	315.6988	221.3103
D1	312.3989	230.0365	mackey	315.4807	230.1314
Djones	315.6676	237.8043	plasma	312.5775	223.6647
Ecg	317.1134	232.7189	qperiodic2	318.302	239.3443
Eeg	312.8058	220.5422	qperiodic3	317.4853	243.7349
Elnino	317.4214	245.1368	rossler	316.6886	220.322
henon	314.5757	227.7612	SP500	315.6374	237.043
hivdna	315.3482	232.6521	Sine	317.4877	219.0768
humandna	316.203	247.3116	Star	312.5734	219.2526
ikeda	313.8854	230.1964	Tent	312.8422	251.1903
kobe	314.9108	225.7457	vanderpol	310.5083	219.0846
laser	313.1771	231.8879	whitenoise	313.7753	230.8806

5. Conclusions

We have analyzed several complex and chaotic time series with two relatively recent analysis techniques, the wavelets and the empirical mode decomposition. We observed results in a time-amplitude domain and in a frequency-time-energy domain to compare their behavior resolution, and then synthesized the data from the components to measure the accuracy of the reconstruction they offer.

The EMD is a step further in the classical signal analysis and the resulting decomposition helps to identify underlying structures and trends however with large data sets its complexity and the fact that it needs from ten to fifteen times the size of the original data set can pose a problem. The wavelet analysis is better suited for large amount of streaming data as it has a fast algorithm and only needs a buffer with the same size of the filters and the resulting coefficients have the same length of the original data.

On the other hand, if one wants to synthesize a signal after some processing in the components, the reconstruction is much simpler with the EMD as one must only sum all the IMFs while the inverse discrete wavelet transform (IDWT) involves filtering and dyadic upsampling in each level.
The EMD is another example where an intuitive computational solution provides better results than an analytical one. Regrettably as of today, the EMD is defined only for one-dimensional data.

Although the EMD has no mathematical proof of orthogonality, the reconstructed time series had a better signal to noise ratio than the ones reconstructed from DWT.

Perhaps it is not important which technique is better suited for a given problem but that we have another useful representation of data which gives more chances to obtain interpretations of such data and to understand phenomena.

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